

Calcul infinitésimal

Exercice 1

$$\forall x \in \mathbb{R} \setminus \{-1, 0, 2\}, f_1'(x) = \frac{-x^4 + 6x^3 - 17x^2 + 8x + 8}{(x^3 - x^2 - 2x)^2}$$

$$\forall x \in \bigcup_{k=0}^{+\infty}]2k\pi, \pi + 2k\pi[, f_2'(x) = \frac{1}{\tan(x)} + \frac{\cos(\ln(x))}{x}$$

$$\forall x \in \mathbb{R}_+^*, f_3'(x) = \left(-\sin(x) \ln(1 + \sqrt{x}) + \frac{\cos(x)}{2\sqrt{x}(1 + \sqrt{x})} \right) (1 + \sqrt{x})^{\cos(x)}$$

$$\forall x \in \mathbb{R} \setminus \left\{ \frac{\pi}{8} + k\frac{\pi}{4}, k \in \mathbb{Z} \right\}, f_4'(x) = \frac{x e^{3x}}{\cos^2(4x)} ((3x + 2) \sin(4x) \cos(4x) + 4x)$$

$$\forall x \in]1, +\infty[, f_5'(x) = \left[(2x + 1) \ln \left(\frac{\ln(x)}{x^2 + 2} \right) + (x^2 + x + 1) \left(\frac{1}{x \ln(x)} - \frac{2x}{x^2 + 2} \right) \right] \left(\frac{\ln(x)}{x^2 + 2} \right)^{x^2 + x + 1}$$

$$\forall x \in \mathbb{R} \setminus \{-1, 1\}, f_6'(x) = \frac{-2x}{x^4 + 1}$$

Exercice 2

$$\frac{\partial f_1}{\partial x}(x, y) = 8x(x^2 - y^3)^3 \quad \text{et} \quad \frac{\partial f_1}{\partial y}(x, y) = -12y^2(x^2 - y^3)^3$$

$$\frac{\partial f_2}{\partial x}(x, y) = e^x \cos(y) - e^{-x} \sin(y) \quad \text{et} \quad \frac{\partial f_2}{\partial y}(x, y) = -e^x \sin(y) + e^{-x} \cos(y)$$

$$\frac{\partial f_3}{\partial x}(x, y) = 3x^2 y^2 - 5y^2 + 6x \quad \text{et} \quad \frac{\partial f_3}{\partial y}(x, y) = 2x^3 y - 10xy - 1$$

$$\frac{\partial f_4}{\partial x}(x, y) = \frac{y}{2x(y-x)} \quad \text{et} \quad \frac{\partial f_4}{\partial y}(x, y) = \frac{-1}{2(y-x)}$$

$$\frac{\partial f_5}{\partial x}(x, y) = \frac{1}{y} \tan\left(\frac{y}{x}\right) - \frac{1}{x \cos^2\left(\frac{y}{x}\right)} \quad \text{et} \quad \frac{\partial f_5}{\partial y}(x, y) = -\frac{x}{y^2} \tan\left(\frac{y}{x}\right) + \frac{1}{y \cos^2\left(\frac{y}{x}\right)}$$

$$\frac{\partial f_6}{\partial x}(x, y, z) = \frac{yz(y^2 + z^2)}{(x^2 + y^2 + z^2)^{3/2}}, \quad \frac{\partial f_6}{\partial y}(x, y, z) = \frac{xz(x^2 + z^2)}{(x^2 + y^2 + z^2)^{3/2}}$$

$$\text{et} \quad \frac{\partial f_6}{\partial z}(x, y, z) = \frac{xy(x^2 + y^2)}{(x^2 + y^2 + z^2)^{3/2}}$$

Exercice 3

$$\forall x \in \mathbb{R}, F_1(x) = -\frac{\cos^4(x)}{4} + C$$

$$\forall x \in \mathbb{R}_+^*, F_2(x) = \frac{(\ln(x))^3}{3} + C$$

$$\forall x \in \mathbb{R}_+^*, F_3(x) = \frac{4}{3} (\sqrt{x} + 1)^{3/2} + C$$

$$\forall x \in]0, 1[, F_4(x) = \ln(-\ln(x)) + C_1 \quad \text{et} \quad \forall x \in]1, +\infty[, F_4(x) = \ln(\ln(x)) + C_2$$

$$\forall x \in \mathbb{R}_+^*, F_5(x) = \frac{2}{7} x^{7/2} + \frac{4}{5} x^{5/2} + 2x^{3/2} + 8x^{1/2} + C$$

$$\forall x \in \mathbb{R}, F_6(x) = \frac{\sqrt{2}}{2} \arctan\left(\frac{\sqrt{2}}{2} x\right) + C$$

Exercice 4

$$\forall x \in \mathbb{R}, F_1(x) = \frac{1}{4} (10x - 11)e^{2x} + C$$

$$\forall x \in \mathbb{R}, F_2(x) = -x^2 \cos(x) + 2x \sin(x) + 2 \cos(x) + C$$

$$\forall x \in \mathbb{R}, F_3(x) = (x^3 - 4x^2 + 9x - 10)e^x + C$$

$$\forall x \in \mathbb{R}, F_4(x) = \left(\frac{x^2}{\ln(2)} - \frac{2x}{\ln(2)^2} + \frac{2}{\ln(2)^3} \right) 2^x + C$$

$$\forall x \in \mathbb{R}, F_5(x) = -\frac{\ln(x)}{2x^2} - \frac{1}{4x^2} + C$$

$$\forall x \in \mathbb{R}, F_6(x) = x \ln^3(x) - 3x \ln^2(x) + 6x \ln(x) - 6x + C$$

Exercice 5

$$(E1) \quad y : x \mapsto C e^{3x}$$

$$(E2) \quad y : x \mapsto C e^{-4x}$$

$$(E3) \quad y : x \mapsto C e^{5x} - \frac{1}{5}$$

$$(E4) \quad y : x \mapsto C e^{-7x/6} + \frac{8}{7}$$

$$(E5) \quad y : x \mapsto C e^{x/2} + 1$$

Exercice 6

$$(E1) \quad y : x \mapsto Ae^{2x} + Be^{3x}$$

$$(E2) \quad y : x \mapsto e^x(A \cos(2x) + B \sin(2x))$$

$$(E3) \quad y : x \mapsto e^{2x}(A \cos(x) + B \sin(x)) + \frac{1}{5}$$

$$(E4) \quad y : x \mapsto (Ax + B)e^{-2x} + \frac{1}{2}$$

$$(E5) \quad y : x \mapsto A + Be^{x/2} + x$$

Exercice 7

1. $y : x \mapsto 3e^{2(1-x)/3} + 2$;

2. $y : x \mapsto e^x(\sin(2x) - 2 \cos(2x)) + 2$

3. $y : x \mapsto \frac{1}{3(e^{2/\sqrt{3}} - 1)} \left((6e^{1/\sqrt{3}} - 5) e^{-\sqrt{3}x + \frac{1}{\sqrt{3}}} + (5e^{1/\sqrt{3}} - 6) e^{\sqrt{3}x} \right) - \frac{5}{3}$;

4. $y : x \mapsto \frac{1}{9} (1 - (1 + 3x)e^{-3x})$;

5. il n'existe pas de solution car sinon $1 = y(0) + y'(0) + y''(0) = 1 + 1 + 1 = 3$, ce qui est absurde.

Exercice 8

Déterminer des primitives de chacune des fonctions suivantes à l'aide d'un changement de variable.

$$\forall x \in \mathbb{R}, F_1(x) = \frac{\sqrt{2}}{2} \arctan\left(\frac{\sqrt{2}}{2}x\right) + C$$

$$\forall x \in \mathbb{R}, F_2(\theta) = \tan\left(\frac{\theta}{2}\right) + C$$

$$\forall x \in \mathbb{R}, F_3(x) = \frac{1}{2} \ln(x^2 + x + 1) - \frac{\sqrt{3}}{3} \arctan\left(\frac{\sqrt{3}}{3}(2x + 1)\right) + C$$