Corrigé de la feuille de TD nº 11

Calculs de dérivées, de primitives et d'intégrales

Exercice 1

$$\forall x \in \mathbb{R} \setminus \{-1, 0, 2\},\ f_1'(x) = \frac{-x^4 + 6x^3 - 17x^2 + 8x + 8}{(x^3 - x^2 - 2x)^2}$$

$$\forall x \in \bigcup_{k=0}^{+\infty}]2k\pi, \pi + 2k\pi[,\ f_2'(x) = \frac{1}{\tan(x)} + \frac{\cos(\ln(x))}{x}$$

$$\forall x \in \mathbb{R}_+^\star,\ f_3'(x) = \left(-\sin(x)\ln\left(1+\sqrt{x}\right) + \frac{\cos(x)}{2\sqrt{x}(1+\sqrt{x})}\right)(1+\sqrt{x})^{\cos(x)}$$

$$\forall x \in \mathbb{R} \setminus \{\frac{\pi}{8} + k\frac{\pi}{4}, k \in \mathbb{Z}\},\ f_4'(x) = \frac{xe^{3x}}{\cos^2(4x)}\left((3x+2)\sin(4x)\cos(4x) + 4x\right)$$

$$\forall x \in]1, +\infty[,\ f_5'(x) = \left[(2x+1)\ln\left(\frac{\ln(x)}{x^2+2}\right) + (x^2+x+1)\left(\frac{1}{x\ln(x)} - \frac{2x}{x^2+2}\right)\right]\left(\frac{\ln(x)}{x^2+2}\right)^{x^2+x+1}$$

$$\forall x \in \mathbb{R} \setminus \{-1, 1\},\ f_6'(x) = \frac{-2x}{x^4+1}$$

Exercice 2

$$\frac{\partial f_1}{\partial x}(x,y) = 8x(x^2-y^3)^3 \quad \text{et} \quad \frac{\partial f_1}{\partial y}(x,y) = -12y^2(x^2-y^3)^3$$

$$\frac{\partial f_2}{\partial x}(x,y) = e^x\cos(y) - e^{-x}\sin(y) \quad \text{et} \quad \frac{\partial f_2}{\partial y}(x,y) = -e^x\sin(y) + e^{-x}\cos(y)$$

$$\frac{\partial f_3}{\partial x}(x,y) = 3x^2y^2 - 5y^2 + 6x \quad \text{et} \quad \frac{\partial f_3}{\partial y}(x,y) = 2x^3y - 10xy - 1$$

$$\frac{\partial f_4}{\partial x}(x,y) = \frac{y}{2x(y-x)} \quad \text{et} \quad \frac{\partial f_4}{\partial y}(x,y) = \frac{-1}{2(y-x)}$$

$$\frac{\partial f_5}{\partial x}(x,y) = \frac{1}{y}\tan\left(\frac{y}{x}\right) - \frac{1}{x\cos^2\left(\frac{y}{x}\right)} \quad \text{et} \quad \frac{\partial f_5}{\partial y}(x,y) = -\frac{x}{y^2}\tan\left(\frac{y}{x}\right) + \frac{1}{y\cos^2\left(\frac{y}{x}\right)}$$

$$\frac{\partial f_6}{\partial x}(x,y,z) = \frac{yz(y^2+z^2)}{(x^2+y^2+z^2)^{3/2}}, \quad \frac{\partial f_6}{\partial y}(x,y,z) = \frac{xz(x^2+z^2)}{(x^2+y^2+z^2)^{3/2}}$$

$$\text{et} \quad \frac{\partial f_6}{\partial z}(x,y,z) = \frac{xy(x^2+y^2)}{(x^2+y^2+z^2)^{3/2}}$$

Exercice 3

$$\forall x \in \mathbb{R},\ F_1(x) = -\frac{\cos^4(x)}{4} + C$$

$$\forall x \in \mathbb{R}_+^\star,\ F_2(x) = \frac{(\ln(x))^3}{3} + C$$

$$\forall x \in \mathbb{R}_+^\star,\ F_3(x) = \frac{4}{3}\left(\sqrt{x}+1\right)^{3/2} + C$$

$$\forall x \in]0,1[,\ F_4(x) = \ln(-\ln(x)) + C_1 \quad \text{et} \quad \forall x \in]1,+\infty[,\ F_4(x) = \ln(\ln(x)) + C_2$$

$$\forall x \in \mathbb{R}_+^\star,\ F_5(x) = \frac{2}{7}x^{7/2} + \frac{4}{5}x^{5/2} + 2x^{3/2} + 8x^{1/2} + C$$

$$\forall x \in \mathbb{R},\ F_6(x) = \frac{\sqrt{2}}{2}\arctan\left(\frac{\sqrt{2}}{2}x\right) + C$$

Exercice 4

$$\begin{aligned}
&\forall x \in \mathbb{R},\ F_1(x) = \frac{1}{4}(10x-11)e^{2x} + C\\
&\forall x \in \mathbb{R},\ F_2(x) = -x^2\cos(x) + 2x\sin(x) + 2\cos(x) + C\\
&\forall x \in \mathbb{R},\ F_3(x) = (x^3 - 4x^2 + 9x - 10)e^x + C\\
&\forall x \in \mathbb{R},\ F_4(x) = \left(\frac{x^2}{\ln(2)} - \frac{2x}{\ln(2)^2} + \frac{2}{\ln(2)^3}\right)2^x + C\\
&\forall x \in \mathbb{R},\ F_5(x) = -\frac{\ln(x)}{2x^2} - \frac{1}{4x^2} + C\\
&\forall x \in \mathbb{R},\ F_6(x) = x\ln^3(x) - 3x\ln^2(x) + 6x\ln(x) - 6x + C
\end{aligned}$$

Exercice 5

$$I_1 = \frac{\pi}{4}, \quad I_2 = \frac{\ln(2)}{2} \quad \text{et} \quad I_3 = \frac{4-\pi}{4}$$

Exercice 6

$$\int_a^b \frac{\mathrm{d}t}{(t+a)(t+b)} = \frac{2\ln(a+b) - \ln(4ab)}{b-a}$$

Exercice 7

$$\begin{aligned}
I_1 &= \frac{3\ln(3)^3 - 9\ln(3)^2 + 18\ln(3) - 12}{\ln(3)^4}\\
I_2 &= \frac{2e^3+1}{9}\\
I_3 &= \frac{\pi(e+1)\sqrt{e}}{\pi^2+1}
\end{aligned}$$

Exercice 8

$$\begin{aligned}
I_1 &= 2\pi\\
I_2 &= \frac{\pi(e+1)}{\pi^2+1}\\
I_3 &= 1
\end{aligned}$$